Asymptotic Theory of Propeller Noise—Part I: Subsonic Single-Rotation Propeller

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Asymptotic expressions for the harmonic amplitudes and phases of the far-field acoustic pressure generated by a single-rotation propeller operating at subsonic tip relative Mach number are presented. These expressions are found from asymptotic approximations to integrals of the steady-loading distribution and of the blade thickness distribution over the surface of one blade, under the assumption that the number of blades *B* is large (but the harmonic number *m* is arbitrary). The asymptotics demonstrate rigorously that in this limit the noise of subsonic propellers is entirely tip generated, and described by very simple formulas giving explicit dependence on harmonic number, Mach number, and radiation angle. Excellent agreement is found between the asymptotic prediction and full numerical evaluation of the acoustic field (and between the latter and experimental data taken by Rolls-Royce in flyovers of a Gannet aircraft). Numerous trends and observations noted in the literature are explained by the asymptotic formulas, which are also extended to predict the acoustic benefit of sweep at subsonic conditions.

Nomenclature

b	= airfoil maximum thickness
B	= number of blades
c	= airfoil chord
c_0	= ambient speed of sound
C_L	= lift coefficient
D	= propeller diameter
F_L	= normalized lift distribution
h	= normalized thickness distribution
k_x, k_y	= wave numbers given by Eqs. (3) and (4)
m	= harmonic of blade passing frequency
M_r	= section relative Mach number
M_{rt}	= blade tip relative Mach number
M_t	= tip rotational Mach number
M_x	= flight Mach number
$M_0(\theta)$	= Mach number defined by Eqs. (14) and (17)
p	= acoustic pressure
P_m	= normalized harmonic pressure coefficient
r_0	= distance from origin to observer point
s	= sweep, see Fig. 1
S	= source strength, see Eq. (8)
t	= observer time
X	= normalized chordwise coordinate
z	= normalized radial coordinate
β	= see Eq. (11)
θ	= radiation angle from propeller axis to observer point
Λ_t	= blade tip sweep angle
ν	= see Eq. (12)
ρ	= density
ϕ_s	= phase lag due to sweep, Eq. (5)
ψ_0	= circumferential angle from observer to reference position, see Fig. 1
Ψ_L, Ψ_V	= chordwise noncompactness factors, Eqs. (2) and (7)
Ω	= shaft angular speed

Subscript

t	= evaluated at blade tip

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I. Introduction

PROPELLER noise at moderate subsonic speeds is an old subject, and one to which intense theoretical and experimental study has been directed. The subject has taken on new importance in the last 10 years with the development of the propfan[1] and related single- and counter-rotating propellers and fans (shrouded and unshrouded). These developments, together with continued public pressure for reduced aircraft noise levels in communities around airports—as reflected in international, national, and local legislation and restrictions—have made vital the formulation of accurate and versatile noise prediction schemes capable of dealing with a wide range of mechanical and geometrical layouts of propellers and fans.

Much progress has been achieved, perhaps most notably by Hanson[2-4] in a series of papers implementing the basic aeroacoustic formulation known as Ffowcs Williams-Hawkings equation.[5] That equation gives an integral prescription for the sound field, given the space-time distribution of quadrupole sources (mainly associated with nonlinear flow disturbances around high-speed blades), of dipole sources associated with the local forces between blade surface and fluid, and of monopole sources associated with the volume displacement effects of the moving blades. A specification of the motion of the quadrupoles and of the blade surface is also required. The quadrupoles have been considered several times before in the context of propeller noise,[6-8] and the present consensus is that they may be ignored for sufficiently thin and/or swept blades operating away from the transonic regime. This view will be adopted in the present paper (although we shall return to it in subsequent work—and the asymptotic methods of this paper will still be indispensible if the quadrupoles do become important), which will concentrate exclusively on the prediction of steady-loading dipole noise and thickness monopole noise for a single-rotation propeller (SRP).

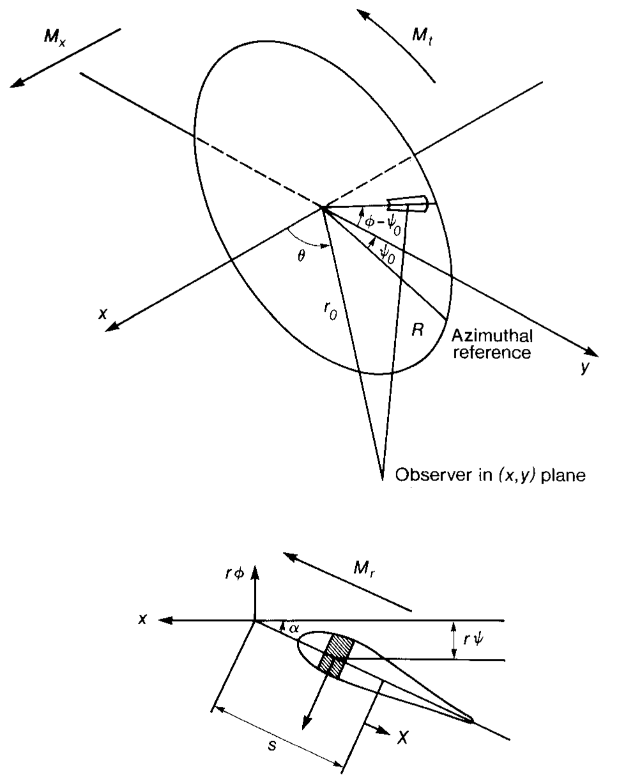

Fig. 1 Propeller geometry: the nominal disk plane and an isolated blade element.

In that case, the loading and thickness inputs to the Ffowcs Williams-Hawkings equation can be regarded as known and given by established steady aerodynamic codes. Then the sound field is determined by a quadrature, which can be put in a very useful form in the *frequency-domain* formulation of Hanson[3] for a regular B-bladed SRP (with full allowance for forward flight of the propeller, and for twist and sweep of each blade, as well as a full distribution of loading and thickness over the surface—span and chord—of each blade); Eqs. (1) and (6) give the Hanson integrals. These formulas have a consistent structure for each source term, monopole and dipole (and also, in fact, for the quadrupole, which is, however, not considered here), and separate clearly the source strength from the acoustic interference effects due to radial and chordwise noncompactness and blade sweep. As will become evident in Sec. III, these formulas are, therefore, ideally suited to our purposes. The adequacy of the formal integral expressions over a wide range of SRP operating conditions has been confirmed many times before[3,9,10] from direct numerical evaluation and comparison with rig or full-scale SRP data; and later we shall give briefly a further comparison of this kind, this time for the rotor-alone tones of a counter-rotation propeller (CRP) during an aircraft flyover—here two special techniques[11,12] are required on the experimental side, which should be of widespread interest.

Accordingly, the main thrust of this and subsequent papers is toward the analytical evaluation of the Hanson integrals by asymptotic procedures. The approximation underlying our asymptotics is that of the "many-bladed propeller" and the formal limit $B \to \infty$. Actually, it is the parameter mB that always appears, where m is the harmonic of blade passing frequency, and we prefer to think of $mB \to \infty$ as being achieved for *all* harmonics, $m = 1, 2 \ldots$, in the many-blade limit. Such an approximation was not previously contemplated in propeller noise theory, probably for a combination of reasons: early propellers had only between two and four blades and $B \gg 1$ may have seemed inapplicable (a mistaken feeling, in our view, certainly for $B = 4$); many modern formulations[7,15,16] stress the time domain rather than the frequency domain, and there $B \gg 1$ is not obviously exploitable; and direct numerical evaluation of the integrals seemed to give accurate results with reasonable computing time. (Other approximations, appropriate only for very low values of mB, have also been considered.[13,14]) Nonetheless, we believe that the asymptotic evaluation of the integrals is worthwhile and important—for all of the reasons given earlier.

We shall show that the resulting expressions retain virtually all of the accurate prediction power of the integrals, and yet give that power in very simple explicit formulas. Furthermore, the formulas give remarkable insight into the sound generation mechanisms. Specifically, they show the exponential domination of the tip region, and that noise control can be achieved only by reduction of the radial gradient of the source at the tip; and they predict the harmonic decay at low and high subsonic speeds and explain the underprediction of the "Gutin point force" approximation. They also show how sweep and noncompactness can be used to reduce the far-field sound. Dependence of the sound field on harmonic number, forward Mach number, tip speed, radiation angle, and parameters specifying the *tip distribution* of loading and thickness will all be found in simple closed form. These dependences are invaluable for preliminary design work and for indicating circumstances under which a full numerical evaluation might be necessary (although, we repeat, the asymptotic formulas themselves will generally be sufficiently accurate for almost all purposes).

II. Formulation and Full Numerical Prediction

The starting point for this investigation is the expression

$$p = \frac{-\rho c_0^2 DB}{8\pi r_0(1 - M_x \cos\theta)} \times \sum_{m=-\infty}^{\infty} \exp\left[\frac{imB\Omega}{(1 - M_x\cos\theta)}\left(t - \frac{r_0}{c_0}\right) + imB\left(\frac{\pi}{2} - \psi_0\right)\right] \times \int_{z_0}^{1} M_r^2 e^{-i\phi_s} J_{mB}\left[\frac{mBM_t\, z\sin\theta}{(1 - M_x\cos\theta)}\right]\left(ik_y\frac{C_L}{2}\right) \times \Psi_L(k_x)\,\mathrm{d}z \tag{1}$$

derived by Hanson[3] for the steady loading noise of a B-bladed SRP. The symbols are defined in the Nomenclature and in Fig. 1. Attention is drawn to the definitions

$$\Psi_L(k_x) = \int_{-\frac{1}{2}}^{\frac{1}{2}} F_L(X) e^{-ik_xX}\,\mathrm{d}X \tag{2}$$

of the chordwise noncompactness factor,

$$k_x = \frac{2mB(c/D)M_t}{(1 - M_x\cos\theta)M_r} \tag{3}$$

$$k_y = \frac{2mB(c/D)(M_r^2\cos\theta - M_x}{(1 - M_x\cos\theta)zM_r} \tag{4}$$

of nondimensional wave numbers k_x and k_y, and

$$\phi_s = \frac{2mB(s/D)M_t}{(1 - M_x\cos\theta)M_r} \tag{5}$$

of a phase contribution due to blade sweep. Other symbols are fairly standard in propeller noise theory, except that z is a normalized spanwise variable ($z = 1$ at the blade tip, $z = z_0$ at the hub) and $M_r = (M_x^2 + z^2M_t^2)^{\frac{1}{2}}$ is the blade section relative Mach number. Equation (1) is also derived by Parry[17] using an alternative method.

For the blade thickness noise, the formula corresponding to Eq. (1) was also derived by Hanson[3] (and again, differently, by Parry[17]) as

$$p = \frac{-\rho c_0^2 DB}{8\pi r_0(1-M_x\cos\theta)} \times \sum_{m=-\infty}^{\infty} \exp\left[\frac{imB\Omega}{(1-M_x\cos\theta)}\left(t-\frac{r_0}{c_0}\right) + imB\left(\frac{\pi}{2}-\psi_0\right)\right] \times \int_{z_0}^{1} M_r^2 e^{-i\phi_s} J_{mB} \left[\frac{mBM_t z\sin\theta}{(1-M_x\cos\theta)}\right]\left(k_x^2\frac{b}{c}\right)\psi_V(k_x)\,\mathrm{d}z \qquad (6)$$

with chordwise noncompactness factor

$$\psi_V(k_x) = \int_{-1/2}^{1/2} h(X)e^{-ik_x X}\,\mathrm{d}X \qquad (7)$$

Before proceeding to asymptotic evaluation of Eqs. (1) and (6), comparisons will be presented of the direct numerical evaluation of Eqs. (1) and (6) with full-scale propeller noise data. Such a comparison has, of course, been made before[2,3,9,10] with generally very favorable results, except at the higher transonic speeds where quadrupole terms might be expected to be significant.[6,7] Nonetheless, the comparison will be made here because the test data were acquired from a 4 × 4 CRP (with differential rotation speeds) on a Fairey Gannet aircraft and will consistently serve as targets for both the SRP and CRP noise prediction schemes to be developed in this and succeeding papers. It is obviously important to be able to assess, as often as possible, the power of full numerical and asymptotic predictions against full-scale data. The comparison will further serve to demonstrate the value and validity of both the split-speed running of the CRP[11] and of the de-Dopplerization technique advocated by Howell et al.[12] Using this technique, Doppler frequency shifting inherent in full-scale

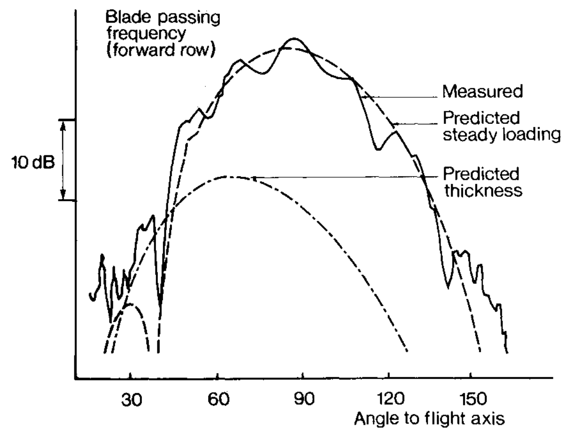

Fig. 2 Gannet measurements vs predictions—first harmonic.

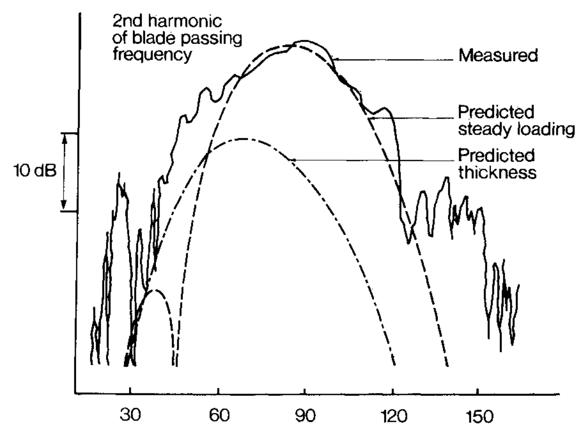

Fig. 3 Gannet measurements vs predictions—second harmonic.

aircraft flyover noise testing can be removed accurately; using a 4% speed differential between the two rows of the counterrotation (4 × 4-bladed) propeller, it is possible to isolate the rotor-alone tones and the interaction tones.

Figures 2 and 3 show the measured and predicted levels, as a function of angle, for the first two harmonics of forward row blade passing frequency. As can be seen, the predictions are dominated by the steady-loading component (as is to be expected at subsonic speeds, and as has been shown by many previous authors,[18-21] and the agreement between measured data and theory is excellent. The loading and thickness distribution inputs for Eqs. (1) and (6), as a function of span radius z, were supplied to Rolls-Royce by Dowty-Rotol, manufacturers of the Gannet propeller. However, since the flight Mach number M_x of the Gannet is very modest (less than 0.3), the chordwise wave number is small for the first few harmonics of blade passing frequency and, hence, the noncompactness factors are approximately equal to 1; i.e., it is not necessary to input the chordwise distributions of loading and thickness but only the spanwise distributions.

III. Asymptotic Approximations

Introduction

In Sec. II, it was seen that linear acoustic theory, using only the thickness monopoles and force dipoles and completely ignoring any quadrupole effects, produces accurate results—at any rate, over the parameter range represented by flight tests of the Gannet aircraft. (In fact, as we observed earlier, quadrupole effects can be ignored for thin/swept blades operating away from the transonic regime.) However, Eqs. (1) and (6) involve numerical integration along the blade span, and the integrand includes a complicated Bessel function as a factor. Furthermore, at conditions where noncompactness effects become important, an additional numerical integration is required along the blade chord. Since results are likely to be required for several harmonics, a range of observer positions, various operating conditions, and different propeller configurations, numerical evaluation of the formulas can become a relatively cumbersome procedure. Therefore, it is useful to have available much simpler approximate formulas from which trends, scaling laws, and possibly even absolute values, can be obtained quickly. This section will address the problem of obtaining suitable approximate formulas.

For conventional subsonic propellers, it has been standard for many years to use the Gutin point force approximation,[13] which was shown by Deming[22] to be accurate for low values of mB. However, several workers[23-25] have shown, for propellers, that the Gutin approximation underpredicts, relative to measured levels, for high values of mB. Other work on hovercraft propellers,[26] on axial flow fans,[27] and on helicopters[28] has also shown that the Gutin formula underpredicts relative to measured data, for high mB. In the last case, comparison was also made with predictions from the Deuce computer program of Dodd and Roper,[29] which includes numerical integration along the blade span and limited chordwise noncompactness effects; the Gutin results were much lower than the computed results results at high values of mB. Since the advanced propellers of interest today have relatively large numbers of blades, the Gutin approximation is therefore likely to prove inaccurate.

Alternative approximations have been derived by Tanna and Morfey[30] for the monopole (thickness) component and by Morfey and Tanna[31] for the dipole (force) component. However, these expressions relate to power spectral density and, therefore, do not retain the full character of expressions for pressure itself, all phase information having been discarded.

A Mach number scaling law for helicopter rotors has been derived by Aravamudan et al.[32] in terms of power spectral density. In their work, however, they neglected the variation in the Bessel function with tip speed, which, as will become evident in what follows, is the most important part of the formulation and of the phenomenon it describes.

Subsonic Operating Conditions—Straight-Bladed Propeller

Start with the case of a straight-bladed propeller operating at low forward speeds. The effects of acoustic chordwise noncompactness are more important at supersonic and high subsonic speeds, since the nondimensional chordwise wave number k_x, defined in Eq. (3), is much larger there. Accordingly, noncompactness effects are considered in a subsequent paper, which discusses propellers operating at *supersonic* conditions (although the results derived there are applicable at both subsonic and supersonic speeds). Therefore, we now set $\phi_s = 0$, $\Psi_L = \Psi_V = 1$.

In order to consider different sources together, we write the harmonic components of the sound field in the form

$$P_m = \int_{z_0}^{1} S(z) J_{mB}\left[\frac{mBM_t\, z \sin\theta}{(1 - M_x \cos\theta)}\right] dz \tag{8}$$

where P_m represents a typical term in the summation in either Eq. (1) or (6) after a factor

$$\frac{-\rho c_0^2 DB}{8\pi r_0 (1 - M_x \cos\theta)} \times \exp\left[\frac{imB\Omega}{(1 - M_x\cos\theta)}\left(t - \frac{r_0}{c_0}\right) + imB\left(\frac{\pi}{2} - \psi_0\right)\right] \tag{9}$$

has been removed. Thus, $S(z)$ represents the variation in source strength with spanwise/radial station [note that $S(z)$ is, of course, different in the loading and thickness cases, and also depends on harmonic, blade number, and the propeller operating parameters], whereas the Bessel function represents the radiation efficiency of sources rotating in the nominal disk plane.

For a propeller operating at subsonic conditions (blade tip relative Mach number less than unity), the argument of the Bessel function will be less than the order. If we consider the Bessel function order mB, representing the product of harmonic and blade number, to be large, we can use the Debye approximation[33]

$$J_{mB}(mB\,\mathrm{sech}\beta) \sim \frac{\exp[mB(\tanh\beta - \beta)]}{(2\pi mB \tanh\beta)^{1/2}} \tag{10}$$

where

$$\mathrm{sech}\beta = \frac{zM_t \sin\theta}{(1 - M_x\cos\theta)} \tag{11}$$

Note that this is quite different from the approximation used by Goldstein[14] and others, in which the argument of the Bessel function is assumed small and the order fixed. We would suggest that that approximation is generally quite inappropriate in propeller noise theory. All experience with Bessel function asymptotics suggests that $mB = 4$ is quite sufficient to permit accurate results using the large mB limit, and that even $mB = 2$ (lowest harmonic of a two-bladed propeller) is better approached through the limit $mB \to \infty$ than through the small-argument, fixed-order limit. We emphasize that it is intended that *all* harmonics ($m = 1, 2, \ldots$) be covered by the same results; thus, our approximation is really that of the *many-bladed* propeller, $B \to \infty$.

Since mB has been assumed large, we see, from the form of Eqs. (10) and (11), that the Bessel function, and hence the integrand of Eq. (8) [because $S(z)$ contains no terms that vary exponentially with mB], increases rapidly toward the tip. Therefore, we can evaluate Eq. (8) using Laplace's method.[34] We put

$$S(z) \sim \bar{S}(1-z)^\nu \quad \text{as} \quad z \to 1 \tag{12}$$

If the source strength is finite at the propeller tips, then $\bar{S} = S(1)$, $\nu = 0$. Equation (8) then reduces to

$$P_m \sim \frac{\bar{S}\exp[mB(\tanh\beta_t - \beta_t)]}{(2\pi mB\tanh\beta_t)^{1/2}} \times \int_{-\infty}^{1} (1-z)^\nu \exp[-mB(1-z)\tanh\beta_t]\, dz \tag{13}$$

where

$$\beta_t = \mathrm{sech}^{-1}\left[\frac{M_t\sin\theta}{(1 - M_x\cos\theta)}\right] \tag{14}$$

and the suffix t refers to the blade tips. We can then evaluate Eq. (13) to give

$$P_m \sim \frac{\bar{S}\exp[mB(\tanh\beta_t - \beta_t)]}{(2\pi mB\tanh\beta_t)^{1/2}} \frac{\nu!}{(mB\tanh\beta_t)^{\nu+1}} \tag{15}$$

Equation (15) is much simpler than the full predictions requiring numerical evaluation and, in addition, retains full dependence on tip rotational Mach number, radiation angle, and harmonic number. The essence of Eq. (15) is that it shows precisely how, under the conditions assumed, SRP noise at subsonic speeds is *tip dominated*.

To show the accuracy of Eq. (15), we compare the full numerical predictions with the asymptotic predictions, for the steady loading noise source, which dominates the sound field at subsonic conditions. (Evaluation of the asymptotic predictions takes, at most, 5% of the CPU time required for the full numerical evaluation of the original integrals.) The values of $\bar{S}$ and ν are obtained by matching Eq. (12) with the tip variation of the radial loading distribution to be used in the full numerical calculation. Figure 4 compares the numerical and the asymptotic steady loading noise solutions for a 12-bladed

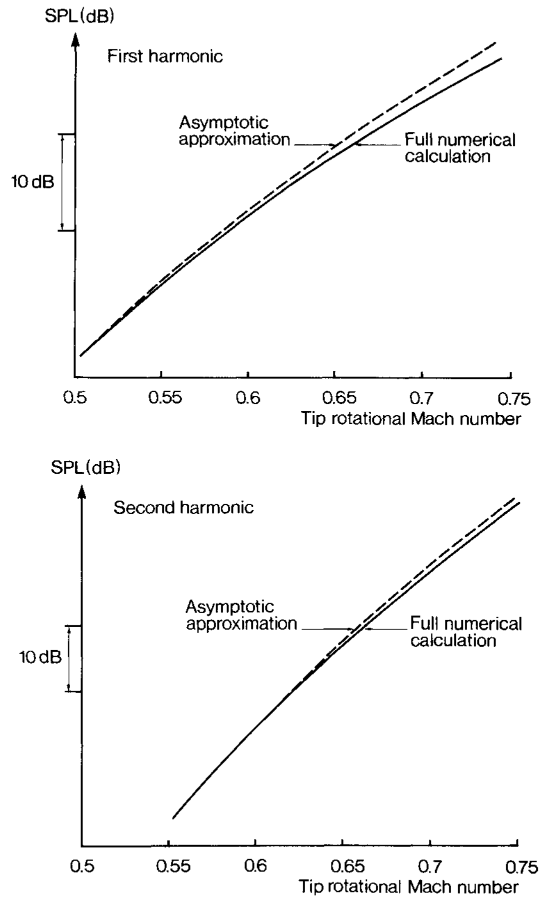

Fig. 4 Numerical and asymptotic predictions for a subsonic, straight-bladed propeller with 12 blades.

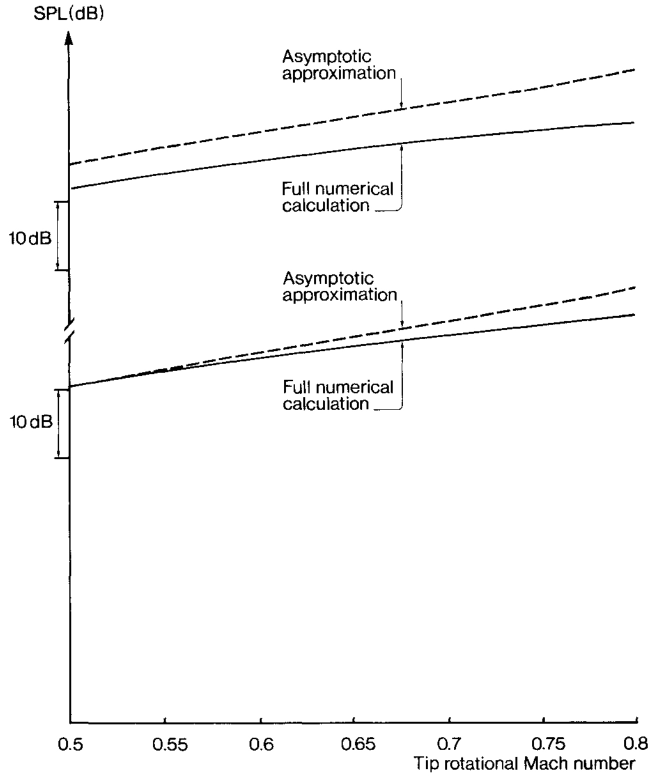

Fig. 5 Numerical and asymptotic predictions for a subsonic, straight-bladed propeller with four blades.

propeller at first and second harmonics of blade passing frequency. The radiation angle was chosen to be 90 deg, since it has been known for many years[35] that, for a propeller operating subsonically, the sound level drops rapidly away from the propeller plane. The figure shows the variation in sound pressure level (SPL) with tip rotational Mach number. It is clear that there is close agreement between the two results across the full range of tip rotational Mach numbers examined, particularly at the second harmonic of blade passing frequency. The decreasing accuracy as M_t approaches unity is to be expected; the underlying approximation [Eq. (10)] is valid for *fixed* $\beta \neq 0$ in the limit $mB \to \infty$ and will fail if a (large) fixed value of mB is chosen and the limit $\beta \to 0$ (corresponding to $M_t \to 1$ when $M_x = 0$ and $\theta = \pi/2$) is taken. A discussion of the asymptotics needed to deal with transonic and supersonic tip speed will follow in Part II of this work.

Figure 5 compares the asymptotic and numerical evaluations of steady-loading noise for a four-bladed propeller at the first harmonic, $m = 1$. The upper diagram refers to a loading distribution that is accurately represented by its tip variation [Eq. (12)] only over the range $0.8 < z < 1$; in the lower diagram the tip distribution is maintained over a larger span, $0.5 < z < 1$. Such a low value of mB is quite unrepresentative of modern propellers, but even so the asymptotic prediction is in fair agreement with the numerical, especially when M_t is moderate.

The asymptotic result [Eq. (15)] can also be used to explain numerous published results, both experimental and theoretical. Trebble,[36-38] for example, found experimentally that at low helical tip speeds the radiated sound field decayed rapidly with the harmonic of blade passing frequency, whereas at higher (subsonic) helical tip speeds there was only a weak decay in the sound field with harmonic number. The same result had also been found earlier by Hubbard and Lassiter[39] (most of whose results were for supersonic tip speeds, but some of which refer to tip rotational Mach numbers of 0.75 and 0.9). To explain this effect, take the dominant term in Eq. (15), which is

$$E = \exp[-mB(\beta_t - \tanh\beta_t)] \tag{16}$$

and rewrite Eq. (14) as

$$\operatorname{sech}\beta_t = 1 + \frac{M_0(\theta) - 1}{1 - M_x \cos\theta} \tag{17}$$

where $M_0(\theta) = M_x \cos\theta + M_t \sin\theta$ is the component of the blade tip total Mach number in the direction, θ, of the observer. The maximum value of $M_0(\theta)$ is, of course, simply the tip Mach number relative to the fluid, M_{rt}, and $M_0(\theta) = M_{rt}$ when the observer lies in the direction of blade motion, $\cos\theta = M_x/M_{rt}$. Attention is confined here to subsonic propellers in the sense that $M_{rt} < 1$, so that $M_0(\theta) < 1$ for all θ. Then when $M_0(\theta)$ is small, β_t is large. Since $\tanh \beta_t < 1$, the argument of the exponential [Eq. (16)] will be large and negative: in fact, $E \sim \exp(-mB\beta_t)$. It is clear that, as the harmonic number m is increased, E will decay very rapidly indeed. However, as $M_0(\theta) \to 1$, β_t becomes small and $(\beta_t - \tanh\beta_t) \sim \beta_t^3/3$, which shows the E will decay only weakly with m: in fact, $E \sim \exp(-mB\beta_t^3/3)$.

An early survey[40] concluded that propeller noise could best be reduced by increasing the number of blades and decreasing the tip speed. The same results were found more recently by Miller and Sullivan,[41] who carried out a parameter study, using a time-domain prediction program, which was aimed at the simultaneous optimization of both noise and performance. These two proposed changes—reducing the tip speed and increasing the blade number—have effects identical to those discussed earlier if we note that increasing the number of blades corresponds to increasing the harmonic number.

In addition, Miller and Sullivan found that if the spanwise (radial) distribution of load was altered so that the inboard loading was increased and the loading near the tip reduced while the total load was maintained constant, then the radiated noise was reduced. Since we know Eq. (15) that most of the noise is generated near the blade tip, it is the reduction in loading *there* that is important. We can see from Eq. (12) that decreasing the loading near the blade tips corresponds to decreasing $\bar{S}$ and/or increasing ν (for the steady-loading component). Equation (15) then shows the precise form of the reduction in the sound field (steady-loading component). Dittmar[42] also used a computer program to look at the effect of moving the loading inboard and found similar results, as did Succi.[43] [Dittmar's work was specifically aimed at supersonic-tip-speed propellers. However, for off-peak observer angles, $M_0(\theta)$ will be less than unity, as is clear from Eq. (17), and in this region our preceding results will be valid.]

In Gutin's original work,[13] the propeller was modeled by an effective source at the radial station $z = 0.8$. In this case, it is still possible to use the asymptotic approximation [Eq. (10)], but instead of Eq. (16), the dominant term will now be given by

$$E = \exp[-mB(\beta_e - \tanh\beta_e)] \tag{18}$$

where

$$\operatorname{sech}\beta_e = 0.8 \operatorname{sech}\beta_t \tag{19}$$

This shows that β_e will be larger than β_t so that Gutin's approximation will overpredict the reduction of noise with an increase of mB, particularly for low tip rotational Mach numbers.

Subsonic Operating Conditions—Swept Propeller

The advanced propellers currently being studied generally incorporate some degree of blade sweep.[9,44] This is mainly for aerodynamic reasons but, in addition, the inclusion of sweep in the blade design may produce acoustic benefits because the signals emitted from different radial stations are partially

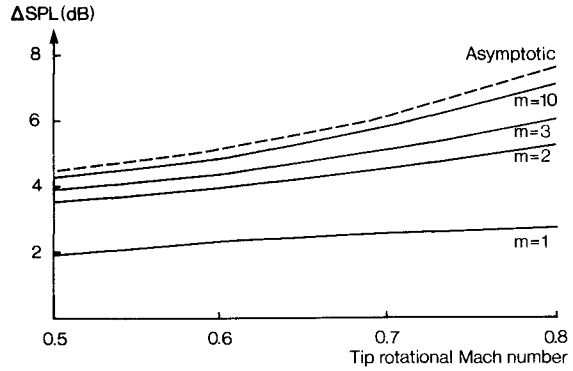

Fig. 6 Numerical and asymptotic predictions of the effect of blade sweep on a subsonic propeller; 12 blades, 50-deg tip sweep.

dephased. Some discussion of this aspect has been given previously by Hanson.[45]

We extend the asymptotic analysis to include the effects of blade sweep. We assume (see later for the proof) that at subsonic operating conditions most of the noise is generated at the blade tips whether the blade is straight or swept. We then linearize the section relative Mach number M_r, and the nondimensional blade sweep s/D (as defined in Fig. 1), about $z = 1$,

$$\frac{s}{D} \sim \frac{s_t}{D} + \frac{(z-1)}{2}\tan\Lambda_t \tag{20}$$

$$M_r \sim M_{rt}\left[1 + (z-1)\frac{M_t^2}{M_{rt}^2}\right] \tag{21}$$

where s_t is the blade tip sweep, Λ_t the blade tip sweep angle, and M_{rt} the tip relative Mach number. This means that the phase ϕ_s, representing the effects of blade sweep, can be approximated by

$$\phi_s \sim \phi_{st} + \frac{2mBM_t}{(1 - M_x\cos\theta)M_{rt}}\left[\frac{\tan\Lambda_t}{2} - \frac{s_t}{D}\frac{M_t^2}{M_{rt}^2}\right](z-1) \tag{22}$$

where ϕ_{st} represents the phase calculated at the blade tips,

$$\phi_{st} = \frac{2mBM_t(s_t/D)}{(1 - M_x\cos\theta)M_{rt}} \tag{23}$$

If we now include this phase term in Eq. (13), we find that

$$P_m \sim \frac{\bar{S}\exp[mB(\tanh\beta_t - \beta_t)]}{(2\pi mB\tanh\beta_t)^{1/2}}\int_{-\infty}^{1}(1-z)^{\nu} \times \exp\left\{mB(z-1)\left[\tanh\beta_t - i\frac{2M_t}{(1-M_x\cos\theta)M_{rt}} \times\left(\frac{\tan\Lambda_t}{2} - \frac{s_t}{D}\frac{M_t^2}{M_{rt}^2}\right)\right]\right\}\,\mathrm{d}z \tag{24}$$

Evaluating this integral and comparing with the result [Eq. (15)] for straight-bladed propellers, we see that the effect of blade sweep on the harmonic components of the acoustic pressure is given by the factor

$$\frac{P_m(\text{swept})}{P_m(\text{straight})} = \left\{1 + \frac{4M_t^2}{M_{rt}^2[(1-M_x\cos\theta)^2 - M_t^2\sin^2\theta]} \times\left(\frac{\tan\Lambda_t}{2} - \frac{s_t}{D}\frac{M_t^2}{M_{rt}^2}\right)^2\right\}^{-(\nu+1)/2} \tag{25}$$

To justify the arguments leading to Eq. (25), we refer to pages 121–125 of Ref. 46, where it is proved that the dominant contribution to the integral

$$\int_a^b \exp[zp(t)]q(t)\,\mathrm{d}t$$

when $|z|$ is large comes from the vicinity of the point $t = t_0$ at which $Re[zp(t)]$ attains its maximum value provided t_0 coincides with an endpoint, a or b. That is the case here; the real part of the argument of the exponential is unaffected by blade sweep and (for subsonic conditions) reaches its maximum at the blade tip. The result that justifies Eq. (15) for unswept blades and Eq. (25) for swept blades is actually Eq. (6.19) of Ref. 46; it shows again that subsonic single-rotor noise is *tip dominated*. (There is, in particular, no significance in any saddlepoint, at which the complex argument of the exponential is stationary, in these circumstances.)

Equation (25) shows that (asymptotically) the noise benefit achieved by incorporating blade sweep into a propeller design is independent of both blade number and harmonic. The predicted noise benefit for a 12-bladed propeller with 50 deg of tip sweep has been calculated using the full numerical calculation [Eq. (1)] and the asymptotic approximation [Eq. (25)]. The results are shown in Fig. 6 for an observer at a 90-deg radiation angle; the tip rotational Mach number varies between 0.5 and 0.8. The numerical calculations are shown for the first, second, third, and tenth harmonics of blade passing frequency. At the first harmonic, the noise reduction, as calculated numerically, is less than the asymptotic prediction across the full range of tip rotational Mach numbers examined. However, at higher harmonics, the numerical results rapidly approach the asymptotic result. This is in accord with intuition, which suggests that the phase oscillations due to sweep weaken the dominance of the tip region at fixed mB, so that a given level of accuracy can be achieved only by increasing mB. Figure 6 indeed shows this behavior.

It has been seen that the asymptotic scheme works well for straight blades, but the reduction ΔSPL associated with sweep is less well predicted for the lower harmonics. Figure 6 shows a reduction of 5 dB for $m = 2$ and $M_t = 0.8$ according to the exact expression, but 7.5 dB according to the asymptotic expression. This represents a significant difference in the overall sound pressure level. However, the much higher spectral components, which dominate perceived noise levels will have a sweep benefit that is much more accurately predicted by the asymptotic expression (which will lead to an overprediction of the sweep benefit by no more than 1 dB for each of those higher components, typically with $m = 4$–8 for current designs). It may also be noted that the nondependence of Eq. (25) on mB (for $mB \gg 1$) is itself an interesting result, and certainly one that can be quickly incorporated into a prediction scheme.

Concluding Remarks

This paper has introduced the asymptotic theory of the noise of a "many-bladed propeller" and has shown how the theory gives simple results that expose the essential mechanisms and yet give accurate predictions when compared with both full numerical evaluations and full-scale flyover data. The next paper in the series will deal with the supersonic single-rotation propeller, for which qualitatively different phenomena arise and which are again clearly and accurately described by the asymptotic theory.

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